WISCONSIN STANDARDS FOR

Mathematics

Wisconsin Department of Public Instruction

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Wisconsin Department of Public Instruction
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Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems. (M)

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Flexibly and efficiently compute with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers. (M)

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables. (M)

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume. (M)

Statistics and Probability

- Develop understanding of statistical variability. (M)
- Summarize and describe distributions. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Content Standards

Ratios and Proportional Relationships (6.RP)

Cluster Statement	Notation	Standard
A. Understand ratio concepts and use ratio reasoning to solve problems. (M)	M.6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
	M.6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* Expectations for unit rates in this grade are limited to non-complex fractions.
	M.6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System (6.NS)

Cluster Statement	Notation	Standard
A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	M.6.NS.A.1	Interpret, represent and compute division of fractions by fractions; and solve word problems by using visual fraction models (e.g., tape diagrams, area models, or number lines), equations, and the relationship between multiplication and division. *For example, create a story context for (2/3) ÷ (3/4) such as "How many 3/4 -cup servings are in 2/3 of a cup of yogurt" or "How wide is a rectangular strip of land with length 3/4 mile and area 2/3 square mile?" Explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.*
B. Flexibly and efficiently compute with multi-digit numbers and find common factors and multiples.	M .6.NS.B.2	Flexibly and efficiently divide multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations.
	M.6.NS.B.3	Flexibly and efficiently add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms based on place value, visual models, the relationship between operations and the properties of operations.
	M.6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
C. Apply and extend previous understandings of numbers to the system of rational numbers. (M)	M.6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

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The Number System (6.NS) (cont'd)

Cluster Statement	Notation	Standard
C. Apply and extend previous understandings of numbers to the system of rational numbers. (M) (cont'd)	M.6.NS.C.6	Understand a rational number as a point on the number line. Extend number lines and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line; find and position pairs of integers and other rational numbers on a coordinate plane.

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The Number System (6.NS) (cont'd)

Cluster Statement	Notation	Standard
C. Apply and extend previous understandings of numbers to the system of rational numbers. (M) (cont'd)	M.6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3 ℃ > -7 ℃ to express the fact that -3 ℃ is warmer than -7 ℃.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write* $\lvert -30 \rvert = 30$ *to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*
	M.6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

The Expressions and Equations (6.EE)

Cluster Statement	Notation	Standard
A. Apply and extend previous understandings of arithmetic to algebraic expressions.	M.6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.
	M.6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*
	M.6.EE.A.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
	M.6.EE.A.4	Identify when two expressions are equivalent (e.g., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

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The Expressions and Equations (6.EE) (cont'd)

Cluster Statement	Notation	Standard
B. Reason about and solve one-variable equations and inequalities.	M.6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
	M.6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
	M.6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
	M.6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
C. Represent and analyze quantitative relationships between dependent and independent variables. (M)	M.6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Geometry (6.G)

Cluster Statement	Notation	Standard
A. Solve real-world and mathematical problems involving area, surface area, and volume. (M)	M.6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
	M.6.G.A.2	Find volumes of right rectangular prisms with fractional edge lengths by using physical or virtual unit cubes. Develop (construct) and apply the formulas $V = l\,w\,h$ and $V = B\,h$ to find volumes of right rectangular prisms in the context of solving real-world and mathematical problems.
	M.6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
	M.6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability (6.SP)

Cluster Statement	Notation	Standard
A. Develop understanding of statistical variability. (M)	M.6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
	M.6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
	M.6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
B. Summarize and describe distributions. (M)	M.6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
	M.6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered and the quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) were given. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Introduction Grade 7

In Grade 7, instructional time should focus on four critical areas: developing understanding of and applying proportional relationships; developing understanding of operations with rational numbers and working with expressions and linear equations; solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and drawing inferences about populations based on samples. Not all content in a given grade is emphasized equally in the standards. Some concepts/skills require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice. “To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade” (Student Achievement Partners 2014).

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

One way to empower students in becoming flexible users of mathematics is to provide authentic real-world opportunities for them to engage in mathematical modeling. The cluster statements that best provide opportunities to implement modeling problem(s) or task(s) are identified with an (M) following the statement. In the middle grades, students are eager to act independently in mathematizing their world. Opportunities for modeling with mathematics are found within their school day as well as in their everyday experiences when they explore messy, complex and non-routine problems. Middle school students might consider plans for ordering pizza for the class. Students would grapple with many decisions and ways of mathematizing the situation as they put together a plan. Students might also consider intentionally open questions, such as "How many pizzas are needed?" or "What toppings should be ordered on the pizza?" At the middle school level, students can play an important role in generating and defining the big modeling questions they would like to address as they attack bigger problems with their growing collection of mathematical tools. This may include considering a big question as a whole class while small groups of students look at a particular aspect of the problem that interest them, making more sophisticated arguments and taking into consideration several constraints at the same time, students critiquing their own work as they report their results, and demonstrating and discussing what happens to their solution when they change an assumption or a particular number (Galluzzo, Levy, Long, and Zbiek 2016, 34).

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems. (M)

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (M)

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (M)

Statistics and Probability

- Use random sampling to draw inferences about a population. (M)
- Draw informal comparative inferences about two populations. (M)
- Investigate chance processes and develop, use, and evaluate probability models. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7 Content Standards

Ratios and Proportional Relationships (7.RP)

Cluster Statement	Notation	Standard
A. Analyze proportional relationships and use them to solve real-world and mathematical problems. (M)	M.7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*
	M.7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.
	M.7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System (7.NS)

Cluster Statement	Notation	Standard
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	M.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line. *a.* Describe situations in which opposite quantities combine to make 0. *For example, if you earn \$10 and then spend \$10, you are left with \$0.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
	M.7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

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The Number System (7.NS) (cont'd)

Cluster Statement	Notation	Standard
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (cont'd)	M.7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

The Expressions and Equations (7.EE)

Cluster Statement	Notation	Standard
A. Use properties of operations to generate equivalent expressions.	M.7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
	M.7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (M)	M.7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.*
	M.7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Flexibly and efficiently apply the properties of operations and equality to solve equations of these forms. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry (7.G)

Cluster Statement	Notation	Standard
A. Draw, construct, and describe geometrical figures and describe the relationships between them.	M.7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
	M.7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
	M.7.G.A.3	Describe the two-dimensional figures that result from slicing three dimensional figures parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids.
B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (M)	M.7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
	M.7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
	M.7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability (7.SP)

Cluster Statement	Notation	Standard
A. Use random sampling to draw inferences about a population. (M)	M.7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
	M.7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
B. Draw informal comparative inferences about two populations. (M)	M.7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
	M.7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
C. Investigate chance processes and develop, use, and evaluate probability models. (M)	M.7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

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Statistics and Probability (7.SP) (cont'd)

Cluster Statement	Notation	Standard
C. Investigate chance processes and develop, use, and evaluate probability models. (M) (cont'd)	M.7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
	M.7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
	M.7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Introduction: Grade 8

In Grade 8, instructional time should focus on three critical areas: formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; grasping the concept of a function and using functions to describe quantitative relationships; analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Not all content in a given grade is emphasized equally in the standards. Some concepts/skills require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice. "To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade" (Student Achievement Partners 2014).

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

One way to empower students in becoming flexible users of mathematics is to provide authentic real-world opportunities for them to engage in mathematical modeling. The cluster statements that best provide opportunities to implement modeling problem(s) or task(s) are identified with an (M) following the statement. In the middle grades, students are eager to act independently in mathematizing their world. Opportunities for modeling with mathematics are found within their school day as well as in their everyday experiences when they explore messy, complex and non-routine problems. Middle school students might consider plans for ordering pizza for the class. Students would grapple with many decisions and ways of mathematizing the situation as they put together a plan. Students might also consider intentionally open questions, such as "How many pizzas are needed?" or "What toppings should be ordered on the pizza?" At the middle school level, students can play an important role in generating and defining the big modeling questions they would like to address as they attack bigger problems with their growing collection of mathematical tools. This may include considering a big question as a whole class while small groups of students look at a particular aspect of the problem that interest them, making more sophisticated arguments and taking into consideration several constraints at the same time, students critiquing their own work as they report their results, and demonstrating and discussing what happens to their solution when they change an assumption or a particular number (Galluzzo, Levy, Long, and Zbiek 2016, 34).

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations. (M)

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities. (M)

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem. (M)
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. (M)

Statistics and Probability

- Investigate patterns of association in bivariate data. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8 Content Standards

The Number System (8.NS)

Cluster Statement	Notation	Standard
A. Know that there are numbers that are not rational, and approximate them by rational numbers.	M.8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and use patterns to rewrite a decimal expansion that repeats into a rational number.
	M.8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

The Expressions and Equations (8.EE)

Cluster Statement	Notation	Standard
A. Work with radicals and integer exponents.	M.8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
	M.8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
	M.8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
	M.8.EE.A.4	Use technology to interpret and perform operations with numbers expressed in scientific notation. Choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).
B. Understand the connections between proportional relationships, lines, and linear equations.	M.8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	M.8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

NOTE: This domain continued on next page.

The Expressions and Equations (8.EE) (cont'd)

Cluster Statement	Notation	Standard
C. Analyze and solve linear equations and pairs of simultaneous linear equations. (M)	M.8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into equivalent forms. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
	M.8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables by graphing and analyzing tables. Solve simple cases represented in algebraic symbols by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions (8.F)

Cluster Statement	Notation	Standard
A. Define, evaluate, and compare functions.	M.8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a numerically valued function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
	M.8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
	M.8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
B. Use functions to model relationships between quantities. (M)	M.8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
	M.8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, continuous or discrete). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry (8.G)

Cluster Statement	Notation	Standard
A. Understand congruence and similarity using physical models, transparencies, or geometry software.	M.8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
	M.8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
	M.8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	M.8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
	M.8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

NOTE: This domain continued on next page.

Geometry (8.G) (cont'd)

Cluster Statement	Notation	Standard
B. Understand and apply the Pythagorean Theorem. (M)	M.8.G.B.6	Justify the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle, and the converse of the Pythagorean theorem.
	M.8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
	M.8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (M)	M.8.G.C.9	Know the relationship among the formulas for the volumes of cones, cylinders, and spheres (given the same height and diameter) and use them to solve real-world and mathematical problems.

Statistics and Probability (8.SP)

Cluster Statement	Notation	Standard
A. Investigate patterns of association in bivariate data. (M)	M.8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	M.8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
	M.8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
	M.8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

High School Standards

This section includes high school specific Standards for Mathematical Practice as well as the Standards for Mathematical Content. These revised state standards (2021) and research recognize that every student needs to have some common outcomes in the first two years of high school, denoted as (F2Y), as well as a third year of further mathematics. Together this completes the 3 years of mathematics required for graduation in Wisconsin state statute. While three years is the minimum, it is common for districts to offer 4 years of mathematics coursework. The (F2Y) standards continue developing students as mathematical thinkers building on their mathematical identity and agency from their work in middle school mathematics. These first two years also prepare students for a variety of options in a third and/or fourth year of mathematics. The third and/or fourth year of mathematics can be designed using a combination of standards not marked as (F2Y). The (+) symbol indicates that the standard is typically one that appears in advanced mathematics courses and is not viewed as mathematics necessary for a three year graduation requirement.

Wisconsin Standards for Mathematics also provides schools/districts with opportunities to make local decisions about curriculum, materials, and assessments. In order to provide guidance for these decisions, efforts have been made to ensure the standards promote educational equity. Examples include:

- Intentionally considering that the standards prepare students for credit-bearing coursework in mathematics at two- and four- year institutions, as well as, for other post-secondary options.
- Identifying standards for a common two year mathematical experience designed to allow for exploration of real-life problems through mathematical modeling as a way to maximize career and college opportunities.

The high school content standards are listed in conceptual categories:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics. A student's engagement with functions, for example, crosses a number of course boundaries, potentially up through and including calculus.

High School Standards for Mathematical Practice

Math Practice 1: Make sense of problems and persevere in solving them.

HS Mathematically proficient high school students analyze givens, constraints, relationships, and goals. They make assumptions where needed to make the problem more clearly articulated. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. Students identify questions to ask and make observations about the situation through notice and wondering strategies. While following a solution plan, they continually ask themselves, "Does this make sense?" They monitor and evaluate their progress and revise their plan as needed. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. High school students might, depending on the context of the problem, transform algebraic expressions to provide them with different information about the situation. They might look at a scatter plot of data or make sense of a situation and decide which family of functions is appropriate to use to model the contextual situation. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph and interpret representations of data, and search for regularity or trends. Mathematically proficient students gain deeper insight into problems by using a different approach, understanding the approaches of others to solve complex problems, and identifying correspondences between different approaches. Mathematically proficient students are engaged in the problem-solving process, do not give up when stuck, and accept that it is acceptable to proceed forward when confronted with confusion and struggle.

Math Practice 2: Reason abstractly and quantitatively.

HS Mathematically proficient high school students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. For example, high school students might work with an exponential function given in symbolic form, but connect how the symbols represent the context of an exponential growth or decay situation. Students are able to manipulate and change the form of the function and reveal different information about the situation based on the numbers stated in the algebraic representation. A student could give a short survey to a sample of students and record the responses in a spreadsheet to compute summary statistics. The computed statistics are then used to gain insight into the larger student population about the question of interest. In addition, students can write explanatory text that conveys their mathematical analyses and thinking, using relevant and sufficient facts, concrete details, quotations, and coherent discussion of ideas. When working with statistics, students need to move from the context to abstract quantities and decide which statistic or representation to use to describe that situation. Students can evaluate multiple sources of information presented in diverse formats (and media) to address a question or solve a problem. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice 3: Construct viable arguments, and appreciate and critique the reasoning of others.

HS Mathematically proficient high school students understand and use stated assumptions, definitions, and previously established results in constructing verbal and written arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples and specific textual evidence to form their arguments. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. While communicating their own mathematical ideas is important, high school students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions that clarify or deepen everyone's understanding, and reconsider their own arguments in response to the collaboration. Mathematically proficient students are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is and why. They can construct formal arguments relevant to specific contexts and tasks. High school students learn to determine domains to which an argument applies. They can present their findings and results to a given audience through a variety of formats such as posters, whiteboards, and interactive materials.

Math Practice 4: Model with mathematics.

HS Mathematically proficient high school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Students that engage in modeling have choice when solving problems. By high school, a student might use geometry to solve a design problem or build a function to describe how one quantity depends on another. Mathematically proficient high school students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that their model will need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as dynamic software, diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. Students will investigate how changes in parameters can result in changes to the model. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, which result in improving the model to better serve its purpose. Students can carry out all phases of the modeling cycle as outlined in the Modeling Conceptual Category. Mathematically proficient high school students also retain the widely applicable techniques they first learned in middle school, such as proportional relationships, rates, and percentages, and apply these techniques as needed to real-world tasks of a complexity appropriate to high school.

Note: Although physical objects and visuals can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, using algebra tiles or an area model to illustrate factoring a quadratic expression would not be an example of practice standard Math Practice 4. Math Practice 4 is about applying math to a problem in context.

Math Practice 5: Use appropriate tools strategically.

HS Mathematically proficient high school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for high school to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students can use slider bars in a dynamic calculator in order to "what-if" a situation and see how the graph of a function changes when the parameters of the equation are changed. They can use a spreadsheet to model change when cells are dynamically linked together and values are changed. Students can analyze graphs of functions and solutions generated using a dynamic graphing device; they also know how to sketch graphs of common functions, choosing this approach over technology when a sketch will suffice (Math Practice 6). They detect possible errors by strategically using estimation and other mathematical knowledge, for example anticipating the general appearance of a graph of a function by identifying the structure of its defining expression (Math Practice 7). They are able to use software or websites to quickly generate data displays that would otherwise be time-consuming to construct by hand (such as histograms, box plots, or simulation models for random sampling). Students use technological tools to explore and deepen their understanding of mathematical concepts and analyze realistic data sets. They use dynamic geometry software to explore geometric transformations of figures using rigid and non-rigid motion. When making mathematical models, students know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data (Math Practice 4). Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.

Math Practice 6: Attend to precision.

HS Mathematically proficient high school students communicate precisely to others both verbally and in writing, adapting their communication to specific contexts, audiences, and purposes. They increasingly use precise language, not only as a mechanism for effective communication, but also as a tool for understanding and solving problems. Describing their ideas precisely helps students understand the ideas in new ways. They use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols that they choose. They are careful about specifying units of measure, labeling axes, defining terms and variables, and calculating accurately and efficiently with a degree of precision appropriate for the problem context. They present logical claims and counterclaims fairly and thoroughly in a way that anticipates the audiences' knowledge, concerns, and possible biases. High school students draw specific evidence from informational sources to support analysis, reflection, and research. They critically evaluate the claims, evidence and reasoning of others and attend to important distinctions with their own claims or inconsistencies in competing claims. Students evaluate the conjectures and claims, data, analysis, and conclusions in texts that include quantitative elements, comparing those with information found in other sources. Diligence and attention to detail are mathematical virtues: mathematically proficient students care that an answer is right; they minimize errors by keeping a long calculation organized; they check their work; they solve the problem another way; they make revisions where appropriate.

Math Practice 7: Look for and make use of structure.

HS Mathematically proficient high school students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, high school students can see the 14 as 2 x 7, and the 9 as 2 + 7. In an equation, high school students recognize that $12 = 3(x-1)^2$ does not require distribution in the expression on the right in order to carry out the process of solving for *x*. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. Students can look closely for patterns and structure that arise in data sets, both large and small. "They use structure to separate the 'signal' from the 'noise' in a set of data—the 'signal' being the structure, the 'noise' being the variability" (Franklin, Bargagliotti, Case, Kader, Scheaffer, and Spangler 2015, 12). Students studying statistical inference will see an inherent structure when developing an estimation interval for a mean or proportion. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. Students make use of structure for a purpose, for example by applying the conclusion $5 – 3(x – y)^2 \leq 5$ in the context of an applied optimization problem. Students will notice that the structure of a quadratic function written symbolically in a-b-c form (standard), vertex form, or factored form will reveal different information about the graph of the function.

Math Practice 8: Look for and express regularity in repeated reasoning.

HS Mathematically proficient high school students notice if calculations are repeated, and look both for general and efficient methods. Noticing the regularity in the way terms sum to zero when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead students to the general formula for the sum of a geometric series. Statistically proficient students maintain oversight of the statistical problem-solving process. They look for a repeatable process that helps to define a given statistic. "Students recognize that probability provides the foundation for identifying patterns in long-run variability, thereby allowing students to quantify uncertainty" (Franklin, Bargagliotti, Case, Kader, Scheaffer, and Spangler 2015, 12). Students recognize that statistics may change value if a random sampling process were repeated. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluating the reasonableness of their intermediate results. When students repeatedly compute products of the form (ax + b) (ax + b) and notice the pattern equals $(a^2x^2 + 2abx + b^2)$, they are looking for and expressing regularity in repeated reasoning. Students change their perspective or view and use what they know. They turn or break down structure to something they know. They solve tasks by solving a sub-problem or smaller version. They might add a line or turn geometric structures so they identify something they have worked with before. By doing this, it helps students move forward and not be stuck. Students use repetition in reasoning as they work with various expressions involving exponents and develop an understanding of the various structures.

High School—Modeling

Mathematical modeling is a messy, open-ended process that is immersed in the real world. It incorporates choice and decision making on the part of the student into the process, and it links classroom mathematics and statistics to everyday life. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, formulate suggestions, and present them to an audience in order to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology may be valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Mathematical modeling is something all students should have access to and be exposed to throughout the mathematics curriculum in all of the courses they take in their high school experience. Mathematical modeling should not be thought of as something that only comes at the end of a unit, but rather a vehicle which can be used to teach new and meaningful mathematics at the beginning and middle of a unit. By engaging students in this process it should disprove the belief that there is only one answer to a problem, and instead encourage them to keep revising their thought process to refine their model in answering questions that are meaningful to them and their community.

A model can be very simple, such as writing the total cost of producing an item as a product of unit price and number of items produced, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or if a two-dimensional disk works well enough for our purposes. We have to invoke choice when deciding how to split the fare amongst three friends heading to different destinations when sharing the same taxi ride. Other situations—modeling a delivery route, developing a production schedule in a factory, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis. Formulating tractable models, representing such models, and analyzing and revising them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

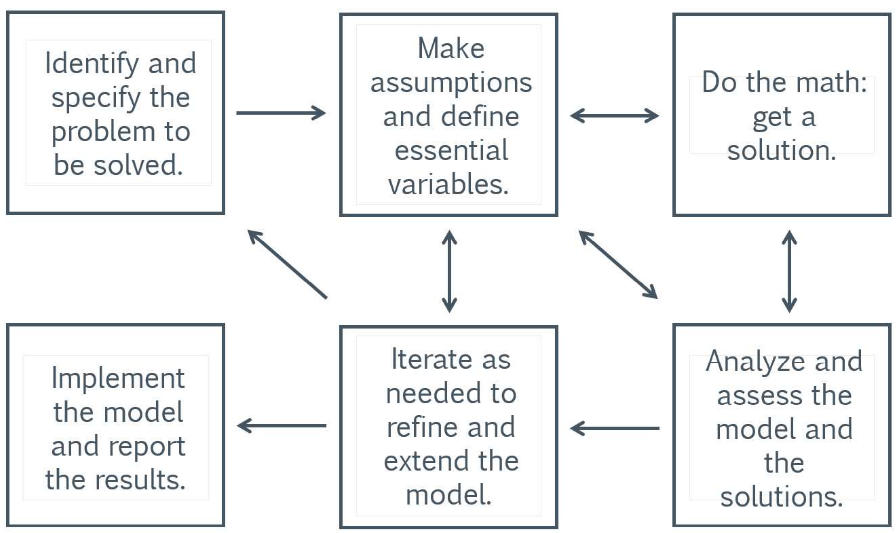
Some examples of modeling situations might include:

- Estimating a cell tower coverage when structural or electrical interference might play a significant factor;
- Minimizing traffic congestion on a busy stretch of road and analyzing how far it will take to drive from one location to another;
- Making the decision if it is better to focus on more three point shots than two point shots in basketball;

- Designing the optimal location for a new school being built in a rural community;
- Analyzing cost savings in driving across town to a different gas station for a cheaper price per gallon of fuel;
- Deciding how to rank a variety of computers based on various attributes for a consumer tips report;
- Designing a path to turnaround an aircraft at an airport; and
- Comparing cost of ownership for two different types of vehicles.

In modeling situations, the models devised depend on a number of factors explored in the following questions. How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves; for example, when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



Source: GAIMME Report, 2016

The basic modeling cycle is summarized in the diagram (Bliss and Libertini 2016, 12-13). We can detail this diagram out starting with the upper left box and proceed clockwise:

1. We identify something in the real world we want to know, do, or understand. The result is a question in the real world.
2. We select 'objects' that seem important in the real world question and identify relations between them. We decide what we will keep and what we will ignore about the objects and their interrelations. The result is an idealized version of the original questions.
3. We translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This formulation is the model. We do the math to see what insights and results we get.
4. We consider: Does it address the problem? Does it make sense when translated back into the real world? Are the results practical, the answers reasonable, the consequences acceptable?
5. We iterate the process as needed to refine and extend our model.
6. For the real world, practical applications, we report our results to others and implement the solution (Bliss and Libertini 2016, 12-13).

Choices, assumptions, and approximations are present throughout the modeling cycle. It is not linear in nature, as the iterative process may have students return to a previous stage before going on to complete the modeling cycle. Functions, ratios and proportions, expressions and equations, descriptive and inferential statistical methods, probabilistic decision, and geometric representations are all important content tools for analyzing mathematical modeling problems. Dynamic graphing applications, spreadsheets, simulation applications, computer algebra systems, interactive applets, and dynamic geometry software are powerful tools that can be used within the mathematical modeling process.

Mathematical modeling is best completed and integrated into other content standards, and more especially in a group of standards within a cluster. Making mathematical models is the Fourth Standard for Mathematical Practice, and specific modeling standards appear throughout the high school content standards indicated by an (M) symbol. The (M) symbol appears next to various clusters of standards throughout all of the conceptual categories in high school mathematics. For instance, in the Functions Conceptual Category you will notice that within the domain of Linear, Exponential, and Quadratic Models there is a cluster of standards called *"Construct and compare linear, quadratic, and exponential models and solve problems,"* followed by (M). The denotation of (M) means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards.

Introduction: Number & Quantity

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number” (1, 2, 3...). Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in three important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Rational and real numbers are number systems utilized in the First Two Years (F2Y) while complex numbers are the new number system introduced in high school courses beyond the first two years.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5. Also, this example also is indicative of a continuous concept that may be introduced as part of a First Two Years (F2Y) and then continued as part of standards in later high school. For instance, students will use the cluster of standards, “*Extend the properties of exponents to rational exponents*”, from this conceptual category when interpreting and building exponential functions in the conceptual category of *Functions* to change the form of the exponential expression to gain further insight on a situation.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. Students have the opportunity to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents as part of the standards work beyond First Two Years (F2Y).

Quantities

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter First Two Years (F2Y) standards that address a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Connection to Mathematical Modeling.

The (M) symbol appears next to various clusters throughout the Number & Quantity conceptual category. For example you will notice the (M) symbol within clusters of standards in the domains of Quantities and Vector and Matrix Quantities. This means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards when addressing them in the curriculum.

Number and Quantity Overview

The Real Number System

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems. (M)

The Complex Number System

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.
- Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities

- Represent and model with vector quantities. (M)
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Real Number System (N-RN)

Cluster Statement	Notation	Standard
A. Extend the properties of exponents to rational exponents.	M.N.RN.A.1 (F2Y)	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents.
	M.N.RN.A.2 (F2Y)	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
B. Use properties of rational and irrational numbers.	M.N.RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities (N-Q)

Cluster Statement	Notation	Standard
A. Reason quantitatively and use units to solve problems. (M)	M.N.Q.A.1 (F2Y)	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	M.N.Q.A.2 (F2Y)	Define appropriate quantities for the purpose of descriptive modeling.
	M.N.Q.A.3 (F2Y)	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System (N-CN)

Cluster Statement	Notation	Standard
A. Perform arithmetic operations with complex numbers.	M.N.CN.A.1	Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real. Understand why complex numbers exist.
	M.N.CN.A.2	(+) Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
	M.N.CN.A.3	(+) Find the conjugate of a complex number; use conjugates to find moduli (absolute values) and quotients of complex numbers.
B. Represent complex numbers and their operations on the complex plane.	M.N.CN.B.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	M.N.CN.B.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus* 2 *and argument* 120°.
	M.N.CN.B.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

NOTE: This domain continued on next page.

The Complex Number System (N-CN) (cont'd)

Cluster Statement	Notation	Standard
C. Use complex numbers in polynomial identities and equations.	M.N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
	M.N.CN.C.8	(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
	M.N.CN.C.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N-VM)

Cluster Statement	Notation	Standard
A. Represent and model with vector quantities. (M)	M.N.VM.A.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
	M.N.VM.A.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	M.N.VM.A.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.

NOTE: This domain continued on next page.

Vector and Matrix Quantities (N-VM) (cont'd)

Cluster Statement	Notation	Standard
B. Perform operations on vectors.	M.N.VM.B.4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v** - **w** as **v** + (-**w**), where -**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	M.N.VM.B.5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of c**v** knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

NOTE: This domain continued on next page.

Vector and Matrix Quantities (N-VM) (cont'd)

Cluster Statement	Notation	Standard
C. Perform operations on matrices and use matrices in applications. (M)	M.N.VM.C.6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
	M.N.VM.C.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
	M.N.VM.C.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
	M.N.VM.C.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
	M.N.VM.C.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	M.N.VM.C.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
	M.N.VM.C.12	(+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Introduction: Algebra

Algebra is an area of high school that connects with both the theoretical and the applied side of mathematics. It strikes balance and complements many of the standards in the Functions conceptual category. It also extends to all of the other conceptual categories as algebraic thinking, syntax, and procedures are used within many of the cluster of standards in those areas. First Two Years (F2Y) standards include much of the work in the Seeing Structure in Expressions domain and the Creating Equations domain, with some work from the Reasoning with Equations and Inequalities domain. During these first two years, students predominantly work with linear, quadratic and exponential expressions and equations. This foundational work connects nicely with the theoretical emphasis on work completed in the Arithmetic with Polynomials & Rational Expressions domain and many of the cluster of standards in the Reasoning with Equations and Inequalities domain that could come after the first two years.

The algebra studied in high school builds on the experiences students had with both expressions and equations throughout K-8 mathematics. Students need to understand the contextual meaning of an expression, while also being able to manipulate it into different forms to reveal various understanding about a situation. They also work with equations and inequalities where they understand the reasoning behind solving and how to rewrite equations in equivalent forms. The form of an algebraic expression and equation is important, depending on the purpose for using it. Different forms may be helpful when trying to better understand what a graphical representation will look like.

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers and arithmetic operations. Students begin seeing and working with numerical and arithmetic expressions early in elementary mathematics. In Kindergarten through Grade 5, students use basic operations of addition, subtraction, multiplication, and division to build expressions. In Grades 6 through 8, the operations expand to using exponents with integers. Through the First Two Years (F2Y) standards, students are able to build upon the basic structure of expressions by adding sophistication through the use of expanded use of exponents and radicals. Within the First Two Years (F2Y) standards, students interpret expressions within context and examine equivalent forms of writing the expression, depending on the situation. As students work in this domain, their work is connected to the cluster of standards of Analyzing functions using different representations in the Function conceptual category. It is here that students will rewrite expressions in equivalent forms to reveal and explain different properties of functions such as quadratics and exponentials. Beyond the First Two Years (F2Y) standards, students write expressions in equivalent form using various techniques, like completing the square. They work with a geometric series to better understand situations such as computing mortgage monthly payments. A spreadsheet or a computer algebra system (CAS) can be used for these situations to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

By extending operations and number systems, students are able to complete more complex algebraic manipulation in attempting to make equivalent expressions that are more appropriate based upon the given context or situation. Algebraic manipulations are governed by the properties of operations that students encounter in Grades 6-8. These properties stem from The Number System and Expressions and Equations domains in Grades 6-8.

Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. Throughout the First Two Years (F2Y), students solve equations and inequalities involving one variable, and create equations involving one or more variables. Techniques involving more complex manipulation are left for standards beyond First Two Years (F2Y). Many of these techniques are listed in the Arithmetic with Polynomials and Rational Expressions domain.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. Students continue their work from 8th grade where they solve systems via inspection to using symbolic and graphical techniques within the First Two Years (F2Y) standards. Beyond those standards, students could use matrices to solve systems of equations.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers. Work with complex solutions is conducted within standards beyond First Two Years (F2Y).

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

When students create equations, they may contain two or more variables. Based on the situation, students decide how algebraic reasoning is used to solve for a variable of interest. These ideas are addressed in First Two Years (F2Y) standards and can be linked to standards that come later, such as work with inverse functions in the Building new functions from existing functions cluster of standards from the Functions conceptual category.

The Algebra category is closely related to the Functions conceptual category. The concept of equivalent expressions can be understood in terms of functions. Two expressions are considered equivalent if they define the same function. An expression in one variable can be viewed as defining a function. Evaluating an expression for a given value is a similar process to finding a function's output for a given input. An equation in two variables can be seen as defining a function, if one of the variables is defined as input and the other output, and if there is just one output for each input.

Connection to Mathematical Modeling

The (M) symbol appears next to various clusters throughout the Algebra conceptual category. For example you will notice the (M) symbol within clusters of standards in the domains of Seeing Structure in Expressions and Creating Equations. This means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards when addressing them in the curriculum.

Algebra Overview

Seeing Structure in Expressions

- Interpret the structure of expressions. (M)
- Write expressions in equivalent forms to solve problems. (M)

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Creating Equations

- Create equations that describe numbers or relationships. (M)

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Algebra Content Standards

Seeing Structure in Expressions (A-SSE)

Cluster Statement	Notation	Standard
A. Interpret the structure of expressions. (M)	M.A.SSE.A.1 (F2Y)	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *For example, in the expression representing height of a projective, -16t2+vt+c recognizing there are three terms in the expression, factors within some of the terms, and coefficients. Interpret within the context the meaning of the coefficient -16 as related to gravity, the factor of v as the initial velocity, and the c-term as initial height.* b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret the expression representing population growth $P(1+r)^n$ as the product of P and a factor not depending on P. Interpret the meaning of the P-factor as initial population, and the other factor as being related to growth rate and a period of time.*
	M.A.SSE.A.2 (F2Y)	Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.*

NOTE: This domain continued on next page.

Seeing Structure in Expressions (A-SSE) (cont'd)

Cluster Statement	Notation	Standard
B. Write expressions in equivalent forms to solve problems. (M)	M.A.SSE.B.3 (F2Y)	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example, if the expression 1.15^t represents growth in an investment account at time t (measured in years), it can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly rate of return is 1.2% based on an annual growth rate of 15%.*
	M.A.SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculating mortgage payments or tracking the amount of an antibiotic in the human body when prescribed for an infection.*

Arithmetic with Polynomials and Rational Expressions (A-APR)

Cluster Statement	Notation	Standard
A. Perform arithmetic operations on polynomials.	M.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
B. Understand the relationship between zeros and factors of polynomials.	M.A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
	M.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
C. Use polynomial identities to solve problems.	M.A.APR.C.4	Prove polynomial identities and use them to describe numerical relationships. *For example, use $(a+20)^2=a^2+40a+400$ to mentally or efficiently square numbers in the 20s. (e.g., $22^2=2^2+2*40+400=484$). Generalize to other double digit numbers. Use $a^2=(A+b)(a-b)+b^2$ and multiples of a*10 to square, e.g., $22^2=(22+12)(22-12)+12^2=340+144=484$. Recognize the visual representation of $(a+2b)^2-a^2=4ab$ as the area of a frame, and find equivalent expressions.*
	M.A.APR.C.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
D. Rewrite rational expressions.	M.A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
	M.A.APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations (A-CED)

Cluster Statement	Notation	Standard
A. Create equations that describe numbers or relationships. (M)	M.A.CED.A.1 (F2Y)	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
	M.A.CED.A.2 (F2Y)	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	M.A.CED.A.3 (F2Y)	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
	M.A.CED.A.4 (F2Y)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange the formula $C = 5/9\ (F\text{-}32)$ so you solve for F.

Reasoning with Equations and Inequalities (A-REI)

Cluster Statement	Notation	Standard
A. Understand solving equations as a process of reasoning and explain the reasoning.	M.A.REI.A.1 (F2Y)	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	M.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
B. Solve equations and inequalities in one variable.	M.A.REI.B.3 (F2Y)	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	M.A.REI.B.4 (F2Y)	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, factoring, and graphing as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

NOTE: This domain continued on next page.

Reasoning with Equations and Inequalities (A-REI) (cont'd)

Cluster Statement	Notation	Standard
C. Solve systems of equations.	M.A.REI.C.5 (F2Y)	Justify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
	M.A.REI.C.6 (F2Y)	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
	M.A.REI.C.7 (F2Y)	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
	M.A.REI.C.8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
	M.A.REI.C.9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

NOTE: This domain continued on next page.

Reasoning with Equations and Inequalities (A-REI) (cont'd)

Cluster Statement	Notation	Standard
D. Represent and solve equations and inequalities graphically.	M.A.REI.D.10 (F2Y)	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
	M.A.REI.D.11 (F2Y)	Explain why the *x-coordinates* of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.
	M.A.REI.D.12 (F2Y)	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Introduction: Functions

Functions are an area of high school that connects heavily with the applied side of mathematics. It strikes balance and complements many of the standards in the Algebra conceptual category. It also extends to Geometry and Statistics & Probability as the notation and concepts are used within the cluster of standards in those areas. A function describes situations where an input value determines another, and extends beyond being just a relationship. Functional relationships should not be thought of only in a numeric sense, as linking a person's social media account to their password would be an example. This example shows the vertical line test should not be the only tool used to decide if a relationship is a function.

Students should also work with functions through multiple representations (symbolic, tabular, graphical, spoken or written words). As they use functions to model meaningful situations, they will move between and within representations to have a better understanding of the phenomena they are trying to understand. In a symbolic representation an algebraic expression like $f(x) = a + bx$; or a recursive rule might be used to relate a restaurant bill to the amount of money left for a tip of X%. Through a tabular representation, a student might see the relationship between weight and height of an individual. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's characteristics. A verbal representation might say, "I'll give you a date from last year, you give me the high temperature in a given city".

Throughout the First Two Years (F2Y) Functions standards, students build on their middle school work involving Ratios & Proportions (6th & 7th grade) and then in 8th grade are introduced to the function concept, though function notation is not required. During 8th grade, they will define, evaluate, and compare functions. They use functions to model relationships between quantities. The construction of these models are focused on linear, but they have experience describing qualitative features and characteristics of both linear and non-linear graphs.

During high school, students will interpret and build functions, which are both domains of study that link together throughout the high school experience. The interpretation of functions is something all students focus on within the clusters of First Two Years (F2Y) standards. They begin to use function notation and better understand the concepts and characteristics associated with them, such as domain and range. Students will explore various families of functions and use them to describe situations that occur in terms of a context. It is here where they will analyze them and move between and within different representations. The work within this domain complements the work within the building functions domain. Within these clusters of standards, students will model a relationship between two quantities and build new functions from existing functions. For instance, a new sub sandwich shop that is opening might want to model the average cost of producing each sandwich by using a cost function and dividing it by a function that represents how many sandwiches are produced. In another situation, a function that models temperature in a house as regulated by a programmable thermostat might be modified if there is a change in desired temperature or time for the heat source to turn on. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

The other area of focus in the Function category is the study of Families of Functions. This links together the Linear, Quadratic, and Exponential domain, with the Trigonometric Functions domain. There is also overlap with other functions that are described in the Algebra category. During the First Two Years (F2Y) cluster of standards, students will focus mainly on Linear, Quadratic and Exponential functions. They will interpret these functions and use them to model real-world phenomena. Appropriate dynamic technology should be used to assist students to see comparisons between the characteristics these families have and the distinctions that occur within parameters and form used to represent in symbolic form, patterns that are noticeable between successive outputs in a table, the structure of their graphs, and appropriate contextual situations that lend themselves to one family of function over another. As with the interpret and build domains, these families of functions standards provide avenues for students to engage in modeling.

It is later in their studies students would experience work with data that is periodic and repeats, such as the height or a rider on a Ferris wheel or the voltage cycle in an alternating current. Both of these could be modeled by a trigonometric function. It is here that students should continue to examine structure and see how the change in one parameter in the symbolic representation would change different characteristics of the function, such as midline or amplitude. Students would explore how a change in one representation would change the other representations. These ideas link back to the clusters of standards that students are exploring within the Building Functions domain. Once again, dynamic technology is appropriate for students to make connections between and within these representations. This entire conceptual category should be one of many areas of inspiration for integrating mathematical modeling problems and tasks during the high school mathematics experience.

Connection to Mathematical Modeling

The (M) symbol appears next to various clusters throughout the Function conceptual category. For example you will notice the (M) symbol within clusters of standards in the domains of Interpreting Functions, Building Functions, Linear, Exponential and Quadratic Models, and Trigonometric Functions. This means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards when addressing them in the curriculum.

Function Overview

Interpreting Functions

- Understand the concept of a function and use function notation.
- Intercept functions that arise in applications in terms of the context. (M)
- Analyze functions using different representations. (M)

Building Functions

- Build a function that models a relationship between two quantities. (M)
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems. (M)
- Interpret expressions for functions in terms of the situation they model. (M)

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions. (M)
- Prove and apply trigonometric identities.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Functions

Interpreting Functions (F-IF)

Cluster Statement	Notation	Standard
A. Understand the concept of a function and use function notation.	M.F.IF.A.1 (F2Y)	Understand that a function from one set, discrete or continuous, (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
	M.F.IF.A.2 (F2Y)	Use function notation, evaluate functions. and interpret statements that use function notation in terms of a context.
	M.F.IF.A.3 (F2Y)	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, in an arithmetic sequence, f(x) = f(x-1) + C or in a geometric sequence, f(x) = f(x-1) * C, where C is a constant.*
B. Interpret functions that arise in applications in terms of context. (M)	M.F.IF.B.4 (F2Y)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
	M.F.IF.B.5	Relate the domain of a function to its graph and find an appropriate domain (discrete or continuous) in the context of the given problem.
	M.F.IF.B.6	Calculate and interpret the average rate of change of a linear or nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

NOTE: This domain continued on next page.

Interpreting Functions (F-IF) (cont'd)

Cluster Statement	Notation	Standard
C. Analyze functions using different representations. (M)	M.F.IF.C.7 M.F.IF.C.7a (F2Y)	Graph functions expressed symbolically and show key features of the graph using an efficient method. a. Graph linear and quadratic functions and show intercepts, maxima, and minima; and exponential functions, showing intercepts and end behavior. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	M.F.IF.C.8 (F2Y)	Write a function defined by an expression in equivalent forms to reveal and explain different properties of the function. a. Use an efficient process to rewrite $f(x) = ax^2+bx+c$ as $f(x) = a(x-h)^2 +k$ or $f(x)=a(x-p)(x-q)$ to determine the characteristics of the function and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions, where t is in years, such as* $y = (1.01)^{12t}$ *is approximately* $y = (1.127)^t$ *, where t is in years, meaning it is a 1% growth rate each month and a 12.7% growth rate each year. Identify percent rate of change in functions, where t is in years, such as* $y = (1.2)^{(t/10)}$ *is approximately* $y = (1.018)^t$ *, meaning it is a 20% growth rate each decade and a 1.8% growth rate each year.*
	M.F.IF.C.9 (F2Y)	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions (F-BF)

Cluster Statement	Notation	Standard
A. Build a function that models a relationship between two quantities. (M)	M.F.BF.A1	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example: The temperature of a cup of coffee can be modeled by combining together a function representing difference in temperature and the actual room temperature, which results in an exponential model. An average cost function can be created by dividing the cost of purchasing n items by the number of n items purchased, which results in a rational function.* c. Work with composition of functions using tables, graphs and symbols. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
	M.F.BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

NOTE: This domain continued on next page.

Building Functions (F-BF) (cont'd)

Cluster Statement	Notation	Standard
B. Build new functions from existing functions.	M.F.BF.B.3	Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* using transformations for specific values of *k* (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
	M.F.BF.B.4	Identify and create inverse functions, using tables, graphs, and symbolic methods to solve for the other variable. *For example: Each car in a state is assigned a unique license plate number and each license plate number is assigned to a unique car; thus there is an inverse relationship. Rearrange the formula C=59(F-32) so you solve for F. You examine a table of values and realize the inputs and outputs are invertible. Two graphs are symmetrical about the line y = x.*
	M.F.BF.B5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models (F-LE)

Cluster Statement	Notation	Standard
A. Construct and compare linear, quadratic, and exponential models and solve problems. (M)	M.F.LE.A.1 (F2Y)	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
	M.F.LE.A.2 (F2Y)	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
	M.F.LE.A.3 (F2Y)	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
	M.F.LE.A.4	For exponential models, express as a logarithm the solution to $abc^t = d$ where a, c, and d are numbers and the base b is 2,10, or e; evaluate the logarithm using technology.
B. Interpret expressions for functions in terms of the situation they model.	M.F.LE.B.5 (F2Y)	Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions (F-TF)

Cluster Statement	Notation	Standard
A. Extend the domain of the trigonometric functions of the unit circle.	M.F.TF.A.1	1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
	M.F.TF.A.2	2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
	M.F.TF.A.3	3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
	M.F.TF.A.4	4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
B. Model periodic phenomena with trigonometric functions. (M)	M.F.TF.B.5	5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
	M.F.TF.B.6	6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
	M.F.TF.B.7	7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
C. Prove and apply trigonometric identities.	M.F.TF.C.8	8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
	M.F.TF.C.9	9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Introduction: Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Within the First Two Years (F2Y) standards, students focus on the domains of congruence, circles, similarity & right triangle trigonometry, and measurement & dimension. As students move from the First Two Years (F2Y), modeling with geometry and geometric properties are addressed at both levels of the standards. Beyond the First Two Year students might work with conics and applications of trigonometry to general triangles. Later in college, some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Connection to Mathematical Modeling

The (M) symbol appears next to various clusters of standards throughout the Geometry conceptual category. For example, you will notice the (M) symbol within clusters in the domains of Similarity, Right Triangles, and Trigonometry, and Geometric Measurement and Dimension. This means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards when addressing them in the curriculum.

Geometry Overview

Congruence

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles. (M)
- Apply trigonometry to general triangles.

Circles

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems. (M)
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry

- Apply geometric concepts in modeling situations. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geometry

Congruence (G-CO)

Cluster Statement	Notation	Standard
A. Experiment with transformations in the plane.	M.G.CO.A.1 (F2Y)	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
	M.G.CO.A.2 (F2Y)	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
	M.G.CO.A.3 (F2Y)	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
	M.G.CO.A.4 (F2Y)	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
	M.G.CO.A.5 (F2Y)	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

NOTE: This domain continued on next page.

Congruence (G-CO) (cont'd)

Cluster Statement	Notation	Standard
B. Understand congruence in terms of rigid motion.	M.G.CO.B.6 (F2Y)	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
	M.G.CO.B.7 (F2Y)	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
	M.G.CO.B.8 (F2Y)	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
C. Prove geometric theorems.	M.G.CO.C.9 (F2Y)	Prove theorems about lines and angles. *Theorems should include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
	M.G.CO.C.10 (F2Y)	Prove theorems about triangles. *Theorems should include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
	M.G.CO.C.11 (F2Y)	Prove theorems about parallelograms. *Theorems should include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

NOTE: This domain continued on next page.

Congruence (G-CO) (cont'd)

Cluster Statement	Notation	Standard
D. Make geometric constructions.	M.G.CO.D.12 (F2Y)	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
	M.G.CO.D.13 (F2Y)	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry (G-SRT)

Cluster Statement	Notation	Standard
A. Understand similarity in terms of similarity trans-formations.	M.G.SRT.A.1 (F2Y)	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	M.G.SRT.A.2 (F2Y)	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	M.G.SRT.A.3 (F2Y)	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
B. Prove theorems involving similarity.	M.G.SRT.B.4 (F2Y)	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
	M.G.SRT.B.5 (F2Y)	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

NOTE: This domain continued on next page.

Similarity, Right Triangles, and Trigonometry (G-SRT) (cont'd)

Cluster Statement	Notation	Standard
C. Define trigonometric ratios and solve problems involving right triangles. (M)	M.G.SRT.C.6 (F2Y)	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
	M.G.SRT.C.7 (F2Y)	Explain and use the relationship between the sine and cosine of complementary angles.
	M.G.SRT.C.8 (F2Y)	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
D. Apply trigonometry to general triangles.	M.G.SRT.D.9	(+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	M.G.SRT.D.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
	M.G.SRT.D.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles (G-C)

Cluster Statement	Notation	Standard
A. Understand and apply theorems about circles.	M.G.C.A.1 (F2Y) [WI.2010. G.C.A.2 and G.C.A.3]	Identify and describe relationships among inscribed angles, radii, and chords. Prove properties of angles for a quadrilateral inscribed in a circle. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
B. Find arc lengths and areas of sectors of circles.	M.G.C.B.2 (F2Y) [WI.2010. G.C.B.5]	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties (G-GPE)

Cluster Statement	Notation	Standard
A. Translate between the geometric description and the equation for a conic section.	M.G.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
	M.G.GPE.A.2	(+) Derive the equation of a parabola given a focus and directrix.
	M.G.GPE.A.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
B. Use coordinates to prove simple geometric theorems algebraically.	M.G.GPE.B.4 (F2Y)	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
	M.G.GPE.B.5 (F2Y)	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
	M.G.GPE.B.6 (F2Y)	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
	M.G.GPE.B.7 (F2Y)	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).

Geometric Measurement and Dimension (G-GMD)

Cluster Statement	Notation	Standard
A. Explain volume formulas and use them to solve problems. (M)	M.G.GMD.A.1 (F2Y)	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
	M.G.GMD.A.2 (F2Y) [WI.2010. G.GMC.A.3]	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
B. Visualize relationships between two-dimensional and three-dimensional objects.	M.G.GMD.B.3 (F2Y) [WI.2010. G.GMD.4]	Identify three-dimensional objects generated by rotations of two-dimensional objects.

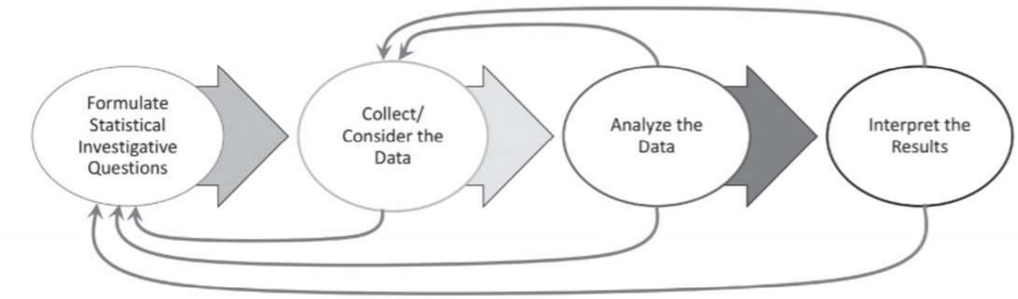
NOTE: This domain continued on next page.

Geometric Measurement and Dimension (G-GMD) (cont'd)

Cluster Statement	Notation	Standard
C. Apply geometric concepts in modeling situations. (M)	M.G.GMD.C.4 (F2Y) [WI.2010. M.G.MG.A.1]	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
	M.G.GMD.C.5 (F2Y) [WI.2010. M.G.MG.A.2]	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
	M.G.GMD.C.6 (F2Y) [WI.2010. M.G.MG.A.3]	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Introduction: Statistics & Probability

First Two Years (F2Y) Statistics & Probability standards should be completed by all high school students. By meeting these standards, students are prepared for further course work addressing standards in the domains of Making Inferences and Justifying Conclusions and Use Probability to Make Decisions. This can lead students to further course work in statistics and data science. Students should be actively engaged in statistics through an investigative process that involves questioning, data collection, analysis and interpretation (Bargagliotti and Franklin 2020, 13).



Decisions or predictions are often based on data—numbers in context. All statistical study should be situated within a context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Throughout the First Two Years (F2Y) Statistics standards, students build on the work they were introduced to in 6th and 8th grades, working with descriptive statistics and linear modeling. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots (dot plots, box plots, and histograms). Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. Students extend the idea of spread from using mean absolute deviation in middle school to interpreting the meaning of standard deviation and what it means for a sample set of data, as well as later in the context of a normal distribution.

Within the Interpret Linear Models cluster of First Two Years (F2Y) standards, students decide if a scatter plot is best modeled by a linear relationship using a correlation coefficient and interpret the meaning of the parameters in the linear model. They examine residuals when comparing observed values to predicted values. Students will also create quadratic and exponential models from scatter plots based on shape and context, which connects to work they did in Functions under the domain of Linear, Exponential and Quadratic Models.

Throughout high school, students further develop their understanding of inferential statistics using simulation, which is grounded in their work from 7th grade. Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data. In critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Throughout the First Two Years (F2Y) probability standards, students build on the work they were introduced to in 7th grade as they work with random processes that can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. During 8th grade, students focused on looking at associations between two categorical variables displayed in a two-way table. Throughout the First Two Years (F2Y) standards their interpretation of these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of different representations such as two-way tables, venn diagrams, and tree diagrams. Also, students use the probability tools they developed during 7th grade and in the First Two Years (F2Y) standards to make decisions about payoffs for a game, analyzing the testing of a medical product, or conduct DNA testing in criminal cases. This is addressed in the "*Use probability* to evaluate outcomes of decisions" cluster of standards.

Technology plays an important role in statistics and probability by making it possible to generate plots (dot plots, box plots, and histograms), regression functions, and correlation coefficients. Statistical software and animations allow students to simulate many possible outcomes in a short amount of time. Students might use bootstrapping techniques or randomization to meet standards within the make inferences and justify conclusions from sample surveys, experiments, and observational studies cluster of standards. Simulation can be used to build conceptual understanding of many of the ideas within inferential statistics, which can lead to a solid foundation for further study in future courses.

Connection to Mathematical Modeling

The (M) symbol appears next to various clusters of standards throughout the Statistics & Probability conceptual category. For example, you will notice the (M) symbol within clusters in the domains of Interpreting Categorical and Quantitative Data, Making Inferences and Justifying Conclusions, Conditional Probability and the Rules of Probability, and Using Probability to Make Decisions. This means that it would be appropriate to implement a modeling problem(s) or task(s) that relates to this group of standards when addressing them in the curriculum.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable. (M)
- Summarize, represent, and interpret data on two categorical and quantitative variables. (M)
- Interpret linear models. (M)

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments. (M)
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (M)

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data. (M)
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems. (M)
- Use probability to evaluate outcomes of decisions. (M)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and appreciate and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Statistics and Probability (SP)

Interpreting Categorical and Quantitative Data (S-ID)

Cluster Statement	Notation	Standard
A. Summarize, represent, and interpret data on a single count or measurement variable. (M)	M.SP.ID.A.1 (F2Y)	Represent data with plots on the real number line (dot plots, histograms, and box plots).
	M.SP.ID.A.2 (F2Y)	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
	M.SP.ID.A.3 (F2Y)	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
	M.SP.ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use statistical packages calculators, spreadsheets, and tables to estimate areas under the normal curve.

NOTE: This domain continued on next page.

Interpreting Categorical and Quantitative Data (S-ID) (cont'd)

Cluster Statement	Notation	Standard
B. Summarize, represent, and interpret data on two categorical and quantitative variables. (M)	M.SP.ID.B.5 (F2Y)	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies as examples of proportionality and disproportionality). Recognize possible associations and trends in the data.
	M.SP.ID.B.6 (F2Y)	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize appropriate families of functions to model.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
C. Interpret linear models (M)	M.SP.ID.C.7 (F2Y)	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
	M.SP.ID.C.8 (F2Y)	Use technology to create a correlation coefficient for a linear fit and then interpret its meaning for the model.
	M.SP.ID.C.9 (F2Y)	Distinguish between correlation and causation. *For example, cities with a higher number of fast food restaurants tend to have more hospitals. While there is a clear correlation (likely driven by population), we cannot conclude that fast food restaurants are causing more hospitals to open. There is a relationship between height and reading level in elementary school children. This is not because changes in height cause better reading ability. Rather as children get older, they get both taller and improve in their reading skills.*

Making Inferences and Justifying Conclusions (S-IC)

Cluster Statement	Notation	Standard
A. Understand and evaluate random processes underlying statistical experiments. (M)	M.SP.IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
	M.SP.IC.A.2	Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (M)	M.SP.IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
	M.SP.IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
	M.SP.IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
	M.SP.IC.B.6	Evaluate reports based on data.

Conditional Probability and the Rules of Probability (S-CP)

Cluster Statement	Notation	Standard
A. Understand independence and conditional probability and use them to interpret data. (M)	M.SP.CP.A.1 (F2Y)	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
	M.SP.CP.A.2 (F2Y)	Understand that two events A and B are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
	M.SP.CP.A.3 (F2Y)	Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
	M.SP.CP.A.4 (F2Y)	Represent data from two categorical variables using two-way frequency tables and/or venn diagrams. Interpret the representation when two categories are associated with each object being classified. Use the representation as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
	M.SP.CP.A.5 (F2Y)	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

NOTE: This domain continued on next page.

Conditional Probability and the Rules of Probability (S-CP) (cont'd)

Cluster Statement	Notation	Standard
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	M.SP.CP.B.6 (F2Y)	Use a representation such as a two-way table or venn diagram to find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
	M.SP.CP.B.7 (F2Y)	Use a representation such as a two-way table or venn diagram to apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
	M.SP.CP.B.8	(+) Use a representation such as a tree diagram to apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.
	M.SP.CP.B.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions (S-MD)

Cluster Statement	Notation	Standard
A. Calculate expected values and use them to solve problems. (M)	M.SP.MD.A.1	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
	M.SP.MD.A.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
	M.SP.MD.A.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
	M.SP.MD.A.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

NOTE: This domain continued on next page.

Using Probability to Make Decisions (S-MD) (cont'd)

Cluster Statement	Notation	Standard
B. Use probability to evaluate outcomes of decisions. (M)	M.SP.MD.B.5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
	M.SP.MD.B.6	Use probabilities to make fair decisions (e.g., drawing for a party door prize where attendees earn one entry to the drawing for each activity they complete, using an electronic spinner to pick a team spokesperson at random from a group, flip a coin to decide which of two friends gets to choose the movie, using a random number generator to select people to include in a sample for an experiment).
	M.SP.MD.B.7	Analyze decisions and strategies using probability concepts (e.g., balancing expected gains and risk, medical product testing, choosing an investment option, deciding when to kick an extra point vs. two point conversion after a touchdown in football).